TRANSFORMATIONS OF GAUSSIAN MEASURES BY STOCHASTIC FLOWS

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Let us consider a solution $\xi(t,\omega,x)$ of the stochastic differential equation

$$d\xi(t,\omega,x)=\sigma(t,\omega)dw_t+b(\xi(t,\omega,x))dt,\quad \xi(0,\omega,x)=x.$$

on the space $\mathbb{R}^n$. It is well known (see [1]) that under the broad assumptions, the transformations U_t on the space $\mathbb{R}^n$ defined by the formula $x\mapsto\xi(t,\omega,x)$, for almost all ω, transport any finite measure with a positive density into an equivalent one. In the recent work [2] an infinite-dimensional generalization of this fact has been obtained. In the cited work, the coefficients σ and b are supposed to possess high regularity, in particular, b must have two bounded derivatives. In our work, an analogous result is proved by a simpler method for a constant coefficient σ and under the only assumption that the drift and its derivative are bounded (the exact formulation is given below). In a more special case, this result has been obtained in the diploma work of the second author.

Let γ be the Gaussian measure on $X=\mathbb{R}^\infty$ that is the direct product of the standard one-dimensional Gaussian measures, let $H=l^2$ be its Cameron–Martin space with the norm $|h|_H=\big(\sum\limits_{n=1}^{\infty}h_n^2\big)^{1/2}$, $h=(h_n)$, and let w_t be a Wiener process on H of the type

$$w_t=\{c_n w_t^{(n)}\}_{n=1}^\infty,\quad \sum_{n=1}^{\infty}c_n^2\le K_0<\infty,$$

where the $w_t^{(n)}$'s are independent one-dimensional Wiener processes, and K with a lower index stands for a nonnegative constant. From now on let $\sigma=1$.

Let a mapping $B\colon X\to H$ be Lipschitzian along H, i.e., $|B(x+h)-B(x)|_H\le C|h|_H$ for all h in H. This ensures (see [3]) that the Gâteaux derivative $D_HB(x)$ along H exists γ a.e. and its operator norm $\|D_HB(x)\|_{L(H)}$ is estimated by C. We need a stronger condition that B be bounded together with the Hilbert–Schmidt norm of its derivative along H: $|B(x)|_H\le K_1$ and $\|D_HB(x)\|_{\mathcal{H}}\le K_2$.

Let us recall that a function $\delta B\in L^1(\gamma)$ is called the divergence of B with respect to γ if

$$\int_X \partial_B f(x)\,\gamma(dx)=-\int_X f(x)\,\delta B(x)\,\gamma(dx),$$

for all smooth real functions f depending on finitely many variables.

In finite dimensions, we have $\delta B(x)=\mathrm{div}B(x)+(B(x),x)$, in the general case the function $\delta B(x)$ is obtained as an $L^2(\gamma)$-limit of appropriate finite-dimensional approximations.

It follows from [3, p. 203] that under the above assumptions, the divergence δB is defined and belongs to $L^2(\gamma)$.

According to [4, p. 288], in our case the divergence satisfies the condition

$$\int_X e^{\varepsilon_0|\delta B(x)|}\gamma(dx)\le M\quad\text{for some positive } \varepsilon_0 \text{ and } M.$$

Let us consider the mapping $U_{st}(\cdot,\omega)\colon\mathbb{R}^\infty\to\mathbb{R}^\infty$ specified by the equation

$$U_{st}(x,\omega)=x+w_t(\omega)-w_s(\omega)+\int_s^t B(U_{sr}(x,\omega))dr,$$

and show that it transports the Gaussian measure γ into an equivalent one. Since B is Lipschitzian along H, this equation has a unique solution for any x.

For every natural number N, we consider the auxiliary transformations

$$\widetilde{U}_{st}^N(x,\omega)=x+\widetilde{w}_t^N(\omega)-\widetilde{w}_s^N(\omega)+\int_s^t \widetilde{B}^N(\widetilde{U}_{sr}^N(x,\omega))dr,$$

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$$\widetilde{V}_{st}^N(x,\omega) = x - \widetilde{w}_t^N(\omega) + \widetilde{w}_s^N(\omega) - \int_s^t \widetilde{B}^N(\widetilde{V}_{rt}^N(x,\omega))dr,$$

where $\widetilde{w}_t^N = \{c_1 w_t^{(1)}, \dots, c_n w_t^{(N)}, 0, 0, \dots\}$ and $\widetilde{B}^N = \{B^{(1)}, \dots, B^{(N)}, 0, 0, \dots\}$.

The solutions $\widetilde{V}_{st}^N$ and $\widetilde{U}_{st}^N$ generate mappings from $\mathbb{R}^N$ into $\mathbb{R}^N$, for which according to [4] we have

$$\gamma \circ (\widetilde{U}_{st}^N)^{-1} = \widetilde{F}_{st}^N \cdot \gamma, \qquad \gamma \circ (\widetilde{V}_{st}^N)^{-1} = \widetilde{G}_{st}^N \cdot \gamma,$$

with the following well-known formulas for the densities:

$$\widetilde{F}_{st}^N = \exp\Big(\int_s^t \delta \widetilde{B}^N\big(\widetilde{V}_{rt}^N(x,\omega)\big)dr - \sum_{n=1}^N \int_s^t c_n (\widetilde{V}_{rt}^N)^{(n)} \circ dw_r^{(n)}\Big),$$

$$\widetilde{G}_{st}^N = \exp\Big(-\int_s^t \delta \widetilde{B}^N\big(\widetilde{U}_{sr}^N(x,\omega)\big)dr + \sum_{n=1}^N \int_s^t c_n (\widetilde{U}_{sr}^N)^{(n)} \circ dw_r^{(n)}\Big).$$

Note that in these formulas we use the Stratonovich stochastic integral, in which, unlike the Ito integral case, the symbol $\circ$ is used in front of the differential sign.

For the proof we need the following two lemmas.

Lemma 1. *Suppose that for every $N \in \mathbb{N}$, P_{st}^N is a continuous function of the variables s and t from $M_{\varepsilon_o} \equiv \{(s,t) \in \mathbb{R}^2 \colon\ 0 \le s \le t \le T, t - s \le \varepsilon_o\}$ to $\mathbb{R}$. Let ε_o, C_1, C_2 be positive constants such that*

$$P_{st}^N \le C_1 + C_2 \int_s^t \int_r^t P_{rz}^N dz dr$$

for all N, s and t, where $(s,t) \in M_{\varepsilon_o}$. Then there exist positive constants $\varepsilon \le \varepsilon_o$ and C such that the functions P_{st}^N are uniformly bounded by the constant C on M_ε.

Proof. Let $\varepsilon = \min\{\varepsilon_0, \sqrt{1/C_2}\}$. Let us fix $N = N_0$. Since $P_{st}^{N_0}$ is continuous on the compact set M_ε, there exist t_0 and s_0 such that $P_{st}^{N_0} \le P_{s_0 t_0}^{N_0}$ on M_ε and

$$P_{s_0 t_0}^{N_0} \le C_1 + C_2 \int_{s_0}^{t_0} \int_r^{t_0} P_{rz}^{N_0} dz dr \le C_1 + C_2 \frac{(t_0 - s_0)^2}{2} P_{s_0 t_0}^{N_0} \le C_1 + C_2 \frac{\varepsilon^2}{2} P_{s_0 t_0}^{N_0} \le$$

$$\le C_1 + \frac{1}{2} P_{s_0 t_0}^{N_0}.$$

Therefore, $P_{s_0 t_0}^{N_0} \le 2C_1$, and since N_0 is arbitrary, then, for all t, s and N from the hypotheses of the lemma, we have $P_{st}^N \le 2C_1$. □

Lemma 2. *Let $\widetilde{F}_{st}^N$ be as defined above. Then there exists $\varepsilon > 0$ such that, for every s and t with $0 \le s \le t \le T, t - s \le \varepsilon$, the family of functions $\{\widetilde{F}_{st}^N(x,\omega)\}$, where $N \in \mathbb{N}$, is uniformly integrable with respect to the measure γ for almost all ω.*

Proof. Let us fix

$$\varepsilon_1 = \min\Big\{\varepsilon_0/8\,, \Big(25\, e\, \sqrt{K_0 + K_1}\,\Big)^{-1}, \ \min_n\Big\{\frac{1}{2\,(12 c_n)^2}\Big\}\Big\},$$

where $\min_n\{\frac{1}{2\,(12c_n)^2}\}$ exists, since $c_n \to 0$ as $n \to \infty$. For the proof of the lemma it suffices to show that

$$I_N = \iint_{\Omega \times X} (\widetilde{F}_{st}^N(x,\omega))^2 \gamma(dx) P(d\omega) \le C, \quad \text{where } C \text{ is independent of } N.$$

In the subsequent expressions the upper index N and the sign $\tilde{}$ are omitted. We have

$$I=\iint \exp\Big(2\int_s^t \delta B(V_{rt}(x,\omega))\,dr-2\sum_n\int_s^t c_nV_{rt}^{(n)}\circ dw_r^{(n)}\Big)\gamma(dx)P(d\omega)$$

$$\le \iint \exp\Big(4\int_s^t \delta B(V_{rt}(x,\omega))\,dr\Big)\gamma(dx)P(d\omega)$$

$$+\iint \exp\Big(-4\sum_n\int_s^t c_nV_{rt}^{(n)}\circ dw_r^{(n)}\Big)\gamma(dx)P(d\omega)\equiv I_1+I_2$$

Let us estimate I_1. We observe that $\exp\Big(\int_s^t \delta B(V_{rt}(x,\omega))dr\Big)=\exp\Big(\int_s^t \varepsilon_1\delta B(V_{rt}(x,\omega))\frac{dr}{\varepsilon_1}\Big)$, which by the Jensen inequality does not exceed $\int_s^t \exp\Big(\varepsilon_1\delta B\big(V_{rt}(x,\omega)\big)\Big)\frac{dr}{\varepsilon_1}$. Let us observe that

$$\int \exp\left(4\varepsilon_1\delta B(V_{rt}(x,\omega))\right)\gamma(dx)=\int \exp\left(4\varepsilon_1\delta B(y)\right)G_{rt}(y,\omega)\gamma(dy), \qquad (1)$$

because U_{rt} is the inverse transformation for V_{rt} and $\gamma\circ U_{st}^{-1}=F_{st}\cdot\gamma$ and $\gamma\circ V_{st}^{-1}=G_{st}\cdot\gamma$.

The Cauchy–Buniakovsky inequality enables one to estimate (1) from above by the expression

$$\sqrt{\int e^{8\varepsilon_1\delta B(y)}\gamma(dy)\int G_{rt}^2(y,\omega)\gamma(dy)}\,.$$

By choosing ε_1 such that $\int e^{8\varepsilon_1\delta B(y)}\gamma(dy)\le\int e^{8\varepsilon_1|\delta B(y)|}\gamma(dy)<M$, and applying the inequality $\sqrt{Q}\le\frac{1}{2}(1+Q)$, we have the following estimate for (1):

$$\frac{\sqrt{M}}{2}\left(1+\int G_{rt}^2(y,\omega)\gamma(dy)\right).$$

Thus, the final estimate for I_1 is this:

$$I_1=\iint \exp\Big(4\int_s^t \delta B(V_{rt}(x,\omega))dr\Big)\gamma(dx)P(d\omega)\le$$

$$\iint\int_s^t \exp\left(4\varepsilon_1\delta B(V_{rt}(x,\omega))\right)\frac{dr}{\varepsilon_1}\,\gamma(dx)P(d\omega)$$

$$\le\frac{\sqrt{M}}{2}+\frac{\sqrt{M}}{2\varepsilon_1}\int_s^t\iint G_{rt}^2(y,\omega)\,\gamma(dy)P(d\omega)\,dr.$$

Let us proceed to estimating

$$I_2=\iint \exp\Big(-4\sum_{n=1}^\infty\Big(\int_s^t(c_nV_{rt}(x,\omega)\circ dw_r^{(n)})\Big)\gamma(dx)\Big)P(d\omega).$$

Plugging $c_nV_{rt}(x,\omega)=c_nx_n-c_n^2(w_t^{(n)}-w_r^{(n)})-c_n\int_r^t B^{(n)}(V_{zt}(x,\omega))\,dz$ in the preceding formula we obtain

$$I_2\le\iint \exp\Big(-12\sum_{n=1}^\infty c_nx_n(w_t^{(n)}-w_s^{(n)})\Big)\,\gamma(dx)P(d\omega)$$

$$+\iint \exp\Big(6\sum_{n=1}^\infty c_n^2\big(w_t^{(n)}-w_s^{(n)}\big)^2\Big)\,\gamma(dx)P(d\omega)$$

$$+\iint \exp\Big(12\sum_{n=1}^\infty c_n\int_s^t\int_r^t B^{(n)}\big(V_{zt}(x,\omega)\big)dz\circ dw_r^{(n)}\Big)\gamma(dx)P(d\omega).$$

Let us denote these integrals by $I_{2,1}$, $I_{2,2}$ and $I_{2,3}$ respectively and estimate $I_{2,3}$, passing from the Stratonovich integral to the Ito integral. We obtain

$$I_{2,3} \leq \iint \exp\Big(24 \sum_n \Big(c_n \int_s^t \int_r^t B^{(n)}(V_{zt})\, dz\, dw_r^{(n)}\Big)\Big)\gamma(dx)P(d\omega)$$

$$+ \iint \exp\Big(-12 \sum_n \Big(c_n \int_s^t \int_r^t \frac{\partial B^{(n)}(V_{zt})}{\partial x_n}\, dz\, dr\Big)\Big)\gamma(dx)P(d\omega)$$

$$\equiv I_{2,3,1} + I_{2,3,2}.$$

For $I_{2,3,1}$ we have

$$I_{2,3,1} = \iint \sum_{k=0}^{\infty} \Big(\frac{1}{k!}\Big(24 \sum_n \Big(c_n \int_s^t \int_r^t B^{(n)}(V_{zt})\, dz\, dw_r^{(n)}\Big)\Big)^k\Big) P(d\omega)\gamma(dx).$$

Let us use the following inequality for the moments of the Ito stochastic integral [5, p. 113]:

$$\mathbb{E}\Big(\int_s^t \theta dw_r\Big)^{2p} \leq 2^p\,(2p-1)^p\,(t-s)^{p-1} \int_s^t \mathbb{E}|\theta|^p dr.$$

By the Cauchy–Buniakovsky inequality one has

$$\mathbb{E}\Big(\int_s^t \theta dw_r\Big)^{2p+1} \leq \Big(\mathbb{E}\Big(\int_s^t \theta dw_r\Big)^{4p+2}\Big)^{\frac{1}{2}} \leq$$

$$\Big(C_p \cdot (t-s)^{2p} \int_s^t \mathbb{E}|\theta|^{2p+1} dr\Big)^{\frac{1}{2}}, \quad \text{where} \quad C_p = (2(4p+1))^{2p+1}.$$

For the first part of $I_{2,3}$ we have

$$I_{2,3,1} \leq 1 + \int \sum_{k=1}^{\infty} \Big(\frac{24^{2k}}{(2k)!}\, 2^k\,(2k-1)^k\,(t-s)^{k-1} \int_s^t E\Big(\sum_n c_n \int_r^t B^{(n)}(V_{zt})\, dz\Big)^k dr\Big)\gamma(dx)$$

$$+ \int \sum_{k=0}^{\infty} \Big(\frac{24^{2k+1}}{(2k+1)!}\Big(C_k \cdot (t-s)^{2k} \int_s^t E\Big(\sum_n c_n \int_r^t B^{(n)}(V_{zt})\, dz\Big)^{2k+1} dr\Big)^{\frac{1}{2}}\Big)\gamma(dx).$$

Now let us estimate the common part of the last summands: $\Big|\int_r^t \sum_n c_n B^{(n)}(V_{zt})\, ds\Big|$. By the assumption, $\sum_n c_n^2 \leq K_0$ and $\sum_n \big(B^{(n)}(y)\big)^2 \leq K_1$, hence

$$\Big|\int_r^t \sum_n c_n B^{(n)}(V_{zt})\, ds\Big| \leq \int_r^t \sum_n \big(c_n^2 + (B^{(n)}(V_{zt}))^2\big) dz \leq (t-s)C.$$

Let us return to estimating $I_{2,3,1}$:

$$I_{2,3,1} \leq 1 + \sum_{k=1}^{\infty} C^k \frac{24^{2k}}{(2k)!} 2^k\,(2k-1)^k (t-s)^{2k} + \sum_{k=0}^{\infty} \frac{24^{2k+1}}{(2k+1)!}\Big(C^{2k+1} C_k \cdot (t-s)^{4k+2}\Big)^{\frac{1}{2}}$$

$$\leq 1 + \sum_{k=1}^{\infty} \frac{C^k}{2k} \frac{24^{2k}\, 2^k}{(2k-1)!} (2k-1)^k \varepsilon_1^{2k} + \sum_{k=0}^{\infty} C^{k+1} \frac{24^{2k+1}}{(2k+1)!}\, 2^{2k+1}\,(2k+1)^{\frac{2k+1}{2}}\, \varepsilon_1^{2k+1}.$$

It remains to estimate the sum by using the inequality $k! \geq (\frac{k}{e})^k$. We obtain

$$I_{2,3,1} \leq 1 + \frac{1}{2e} \sum_{k=1}^{\infty} \frac{(24\, e\, \varepsilon_1)^{2k}\,(2C)^k}{k\,(2k-1)^{k-1}} + \sqrt{C} \sum_{k=0}^{\infty} \frac{(48\, e\, \varepsilon_1 \sqrt{C})^{2k+1}}{(2k+1)^{\frac{2k+1}{2}}} \leq C_{2,3,1}\,.$$

Let us proceed to the second part of $I_{2,3}$. Since $\|DB\|_{\mathcal{H}} \le K_2$, then $\sum_n \Big(\frac{\partial B^{(n)}(V_{zt})}{\partial x_n}\Big)^2 \le q$ and

$$\Big|\sum_n c_n \frac{\partial B^{(n)}(V_{zt})}{\partial x_n}\Big| \le \sum_n c_n^2 \times \sum_n \Big(\frac{\partial B^{(n)}(V_{zt})}{\partial x_n}\Big)^2 \le q\,K_0 < \infty,$$

$$I_{2,3,2} = \iint \exp\Big(-12\sum_n \Big(c_n \int_s^t \int_r^t \frac{\partial B^{(n)}(V_{zt})}{\partial x_n}\,dz\,dr\Big)\Big)\gamma(dx)P(d\omega)$$
$$\le \frac{1}{2}\iint \exp\big(12\,q\,K_0\,t^2\big)\,\gamma(dx)P(d\omega) \le C_{2,3,2}.$$

Now let us estimate $I_{2,1}$ as follows:

$$I_{2,1} = \iint \exp\Big(\sum_{n=1}^{\infty} -12 c_n x_n (w_t^{(n)} - w_s^{(n)})\Big)P(d\omega)\gamma(dx)$$
$$= \iint \prod_n \exp\big(-12 c_n x_n (w_t^{(n)} - w_s^{(n)})\big)P(d\omega)\gamma(dx).$$

Since the processes $w_t^{(n)} - w_s^{(n)}$ are independent, we obtain

$$I_{2,1} = \int \prod_n \int \exp\Big(-12 c_n x_n (w_t^{(n)} - w_s^{(n)})\Big)\,P(d\omega)\gamma(dx).$$

If $\xi \sim N(0,1)$ and $0 < b < 1$, then $\mathbb{E}e^{a\xi} = e^{\frac{a^2}{2}}, \quad \mathbb{E}e^{\frac{b\xi^2}{2}} = (1-b)^{-\frac{1}{2}}$. Hence

$$I_{2,1} = \int \prod_n \mathbb{E}\exp\big(-12 c_n x_n \sqrt{t-s}\,\xi\big)\gamma(dx) = \prod_n \int \exp\Big(\frac{(12c_n)^2(t-s)\,x_n^2}{2}\Big)\gamma(dx)$$
$$= \prod_n \mathbb{E}\exp\Big(\frac{(t-s)(12c_n)^2\xi^2}{2}\Big) = \prod_n \big(1-(t-s)(12c_n)^2\big)^{-\frac{1}{2}}.$$

Since $(t-s)(12c_n)^2 \le 2\varepsilon_1(12c_n)^2 < 1$ and $c_n \to 0$ as $n \to \infty$, then

$$\prod_n \big(1-(t-s)(12c_n)^2\big)^{-\frac{1}{2}} \sim \prod_n \Big(1 + \frac{(t-s)(12c_n)^2}{2}\Big) \sim (t-s)\,K \sum_n c_n^2 < \infty.$$

Hence $I_{2,1} \le (t-s)\,C_{2,1}$. Now let us estimate $I_{2,2}$. We obtain

$$I_{2,2} = \iint \exp\Big(6\sum_{n=1}^{\infty} 24 c_n^2 \big(w_t^{(n)} - w_s^{(n)}\big)^2\Big)\gamma(dx)P(d\omega)$$
$$= \int \exp\Big(6\sum_{n=1}^{\infty} 24 c_n^2 \big(w_t^{(n)} - w_s^{(n)}\big)^2\Big)P(d\omega).$$

Similarly to the estimate for $I_{2,1}$, by using independence of the sequence $w_t^{(n)} - w_s^{(n)}$ we obtain

$$I_{2,2} \le \prod_n \mathbb{E}\exp\Big(\frac{2(t-s)(12c_n)^2\xi^2}{2}\Big) = \prod_n \big(1-2(t-s)(12c_n)^2\big)^{-\frac{1}{2}} \le (t-s)\,C_{2,2}.$$

Thus,

$$I = \iint F_{st}^2(x,\omega)\gamma(dx)P(d\omega) \le C + \frac{\sqrt{M}}{2\varepsilon_1}\int_s^t \iint G_{rt}^2(y,\omega)\gamma(dy)P(d\omega)\,dr.$$

By exactly the same calculations one can estimate

$$J = \iint G_{st}^2(x,\omega)\gamma(dx)P(d\omega) \le C + \frac{\sqrt{M}}{2\varepsilon_1}\int_s^t \iint F_{sr}^2(y,\omega)\gamma(dy)P(d\omega)\,dr,$$

which yields the inequality

$$I = \iint F_{st}^2(x,\omega)\gamma(dx)P(d\omega) \le \widetilde{C} + \frac{M}{4\varepsilon_1^2}\int_s^t \int_r^t \iint F_{rz}^2(y,\omega)\gamma(dy)P(d\omega)\,dz\,dr.$$

It remains to apply Lemma 1 for $P_{st} = \iint F^2_{st}(x,\omega)\gamma(dx)P(d\omega)$, which completes the proof of Lemma 2. $\square$

Theorem 1. *Under the indicated hypotheses, every mapping U_{st} transports the Gaussian measure γ into an equivalent one.*

Proof. Let us apply Lemma 2 and choose $\varepsilon > 0$ such that, for every s, t with $0 \le s \le t \le T, t - s \le \varepsilon$, the family of functions $\{\widetilde{F}^N_{st}(x,\omega)\}$ is uniformly integrable with respect to the measure γ for almost all ω. The sequence of functions $\widetilde{F}^N_{st}(x,\omega)$ converges a.e. to the function

$$F_{st} = \exp\Big(\int_s^t \delta B\big(V_{rt}(x,\omega)\big)dr - \sum_n \int_s^t c_n V^{(n)}_{rt} \circ dw^{(n)}_r\Big) \tag{2}$$

$N \to \infty$. Therefore, as $N \to \infty$, the integrals $\iint_{\Omega\times X}(\widetilde{F}^N_{st}(x,\omega))\gamma(dx)P(d\omega)$ converge to the integral $\iint_{\Omega\times X}(F_{st}(x,\omega))\gamma(dx)P(d\omega)$. From this and pointwise convergence of $\widetilde{U}^N_{st}(x,\omega)$ to $U_{st}(x,\omega)$ we obtain that for all s,t satisfying the conditions $0 \le s \le t \le T, t - s \le \varepsilon$, there holds the equality

$$\gamma \circ U^{-1}_{st} = F_{st} \cdot \gamma.$$

It remains to get rid of the restriction $t - s \le \varepsilon$. Let us use the semigroup property U_{st}: since for all t_1, t_2 and t_3 of the form $0 \le t_1 < t_2 < t_3 \le T$, we have

$$U_{t_1t_3}(x,\omega) = U_{t_2t_3}(U_{t_1t_2}(x,\omega),\omega),$$

then U_{st} can be represented as a composition of at most $[\frac{T}{\varepsilon}]+1$ transformations, each of which transports γ into an equivalent measure. Hence U_{st} also transports the measure γ into an equivalent one, which completes the proof. $\square$

Remark. As it has been shown, for small $|t-s|$, formula (2) expresses the density of the measure transported by the flow with respect to the initial measure. For large $|t-s|$ it may make no sense, although the equivalence of the measures holds.

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References

[1] Kunita H. Stochastic flows of diffeomorphisms. Cambridge University Press, 1984.

[2] Pilipenko A.Yu. Transformations of measures in infinite-dimensional spaces by the flow generated by a stochastic differential equation. Sbornik Math. 2003. V. 194, N 4. P. 85-106.

[3] Bogachev V.I. Gaussian measures. Amer. Math. Soc., Providence, 1998.

[4] Ustunel A.S., Zakai M. Transformation of Measure on Wiener Space. Springer-Verlag, Berlin, 2000.

[5] Krylov N.V. Controlled diffusion processes. Springer, 1977.

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